

# ALGEBRA 2

## Unit 8

The collage features several worksheets with the following titles and content:

- RATIONAL EXPRESSION**: Notes/Examples section defining a rational expression and steps to simplify it: 1. Factor everything that can, 2. Simplify the monomials, 3. Eliminate common binomials. Example:  $\frac{20x^4}{14x^2}$ .
- COMPLEX FRACTIONS**: Notes/Examples section defining a complex fraction and steps to simplify it: 1. Create a single fraction, 2. Set up as a division problem, 3. Simplify! (Factor first if possible). Example:  $\frac{2x}{x^2-12x+27}$ .
- RATIONAL EXPRESSION**: Example problem: "Write an expression in simplest form to represent the area of the rectangle." with a diagram of a rectangle with height  $\frac{x^2-81}{4x^2+36x}$  and width  $\frac{2x}{x^2-12x+27}$ .
- KEY FEATURES of Rational Functions**: Notes/Examples section defining a rational function and identifying key features: x-intercepts, vertical asymptotes, holes. Example:  $f(x) = \frac{x^2-4}{x^2-5x+6}$  with a graph showing a hole at  $(2, 1/3)$  and x-intercepts at  $(-2, 0)$  and  $(4, 0)$ .
- VARIATION Models**: Notes/Examples section defining variation and identifying key features: DIRECT VARIATION ("y is directly proportional to x"), JOINT VARIATION ("y varies jointly as x and z"), and INVERSE VARIATION ("y is inversely proportional to x"). Example:  $y = 2x$ .
- RECIPROCAL Function**: Notes/Examples section defining a reciprocal function and graphing it.

# RATIONAL FUNCTIONS

NOTES • HOMEWORK • QUIZZES • TEST

Created by: ALL THINGS ALGEBRA®

## Unit 8 - Rational Functions: Sample Unit Outline

	<b>TOPIC</b>	<b>HOMEWORK</b>
<b>DAY 1</b>	Simplifying Rational Expressions; Multiplying & Dividing Rational Expressions	HW #1
<b>DAY 2</b>	Adding and Subtracting Rational Expressions	HW #2
<b>DAY 3</b>	<b>Quiz 8-1</b>	None
<b>DAY 4</b>	Complex Fractions	HW #3
<b>DAY 5</b>	Review all Operations	HW #4
<b>DAY 6</b>	Applications	HW #5
<b>DAY 7</b>	<b>Quiz 8-2</b>	None
<b>DAY 8</b>	Graphing Basic Rational Functions (Reciprocal Function)	HW #6
<b>DAY 9</b>	Key Features of Rational Functions ( $x$ -Intercepts, Vertical & Horizontal Asymptotes, Holes)	HW #7
<b>DAY 10</b>	Graphing Rational Functions	HW #8
<b>DAY 11</b>	Graphing Review	HW #9
<b>DAY 12</b>	Solving Rational Equations	HW #10
<b>DAY 13</b>	<b>Quiz 8-3</b>	None
<b>DAY 14</b>	Direct, Inverse, Joint, and Constant Variation	HW #11
<b>DAY 15</b>	Unit 8 Review	Study for Test
<b>DAY 16</b>	<b>UNIT 8 TEST</b>	None

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Topic: \_\_\_\_\_ Class: \_\_\_\_\_

**RATIONAL EXPRESSION**

**SIMPLIFYING Rational Expressions**

**Notes/Examples**

A rational expression is a fraction in which the numerator and denominator both contain fractions.

- Factor
- Cancel
- Eliminate

1.  $\frac{20x^2}{14x^2}$

2.  $\frac{20x^2 + 30x}{4x^2 + 6x^3}$

3.  $\frac{3}{3}$

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Per: \_\_\_\_\_

Algebra 2 Unit 8: Rational Functions

**Quiz 8-1: Simplifying & Operations with Rational Expressions**

Simplify each expression.

1.  $\frac{20x^2 + 30x}{4x^2 + 6x^3}$

2.  $\frac{k^2 + 13k + 40}{k^2 - 2k - 35}$

1. \_\_\_\_\_

2. \_\_\_\_\_

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Topic: \_\_\_\_\_ Class: \_\_\_\_\_

**COMPLEX FRACTIONS**

**Notes/Examples**

- A **complex fraction** is a fraction in which the numerator and denominator both contain fractions.
- To **simplify a complex fraction** means to rewrite as a single fraction with no fractions in the numerator or denominator.

- Create a single fraction in the numerator and denominator.
- Set up as a division problem, then multiply by the reciprocal.
- Simplify! (Factor first if possible.)

**EXAMPLES**


1.  $\frac{\frac{2}{a^2}}{\frac{4}{a}}$

2.  $\frac{\frac{6}{3y}}{\frac{8x^2}{y}}$

**RATIONAL EXPRESSIONS: Applications**

**Example**

1. Write an expression in simplest form to represent the **area** of the rectangle.



$\frac{2x}{x^2 - 12x + 27}$

$\frac{x^2 - 81}{4x^2 + 36x^2}$

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Topic: \_\_\_\_\_ Class: \_\_\_\_\_

**KEY FEATURES of Rational Functions**

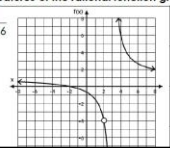
**Notes/Examples**

A **rational function** is a function of the form  $f(x) = \frac{p(x)}{q(x)}$  where  $p(x)$  and  $q(x)$  are polynomial functions. So far, we have looked at the graph of a very basic rational function (a reciprocal function).

Now, we will look at more complex rational functions and their key features: x-intercepts, vertical asymptotes, horizontal asymptotes, and holes. A hole is a point  $(x, y)$  at which there is a break in the graph.

**Identify the key features of the rational function given each graph below.**

1.  $f(x) = \frac{x^2 - 4}{x^2 - 5x + 6}$



x-intercept(s): \_\_\_\_\_

Vertical Asymptote(s): \_\_\_\_\_

Horizontal Asymptote: \_\_\_\_\_

Hole(s): \_\_\_\_\_

**RATIONAL Functions**

**EQUATION FORM:** \_\_\_\_\_

**X-INTERCEPTS:** \_\_\_\_\_

**VERTICAL ASYMPTOTES:** \_\_\_\_\_

**EXAMPLES**

1.  $f(x) = \frac{x^2 - 9}{x + 2}$

x-int: \_\_\_\_\_ Holes: \_\_\_\_\_

Vertical Asymptote: \_\_\_\_\_ D: \_\_\_\_\_

Horizontal Asymptote: \_\_\_\_\_ R: \_\_\_\_\_

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Topic: \_\_\_\_\_ Class: \_\_\_\_\_

**RATIONAL Function**

**Notes/Examples**

A rational function is a function of the form  $f(x) = \frac{p(x)}{q(x)}$  where  $p(x)$  and  $q(x)$  are \_\_\_\_\_

One of the most basic rational functions which is also known as a **reciprocal**.

**RECIPROCAL**

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Topic: \_\_\_\_\_ Class: \_\_\_\_\_

**VARIATION Models**

**Identifying EQUATIONS**

**DIRECT VARIATION Examples**

1.  $y = 2x$

2.  $y = \frac{24}{x}$

3.  $y = 5xz$

4.  $3y = x$

5.  $xy = 36$

6.  $\frac{4}{3}y = xz$

7.  $\frac{y}{x} = \frac{2}{5}$

8.  $2A = bh$

9. If  $y$  is directly proportional to  $x$  and  $y = 28$  when  $x = 7$ , find  $x$  when  $y = 52$ .

10. If  $r$  is directly proportional to  $s^2$  and  $r = 12$  when  $s = 2$ , find  $r$  when  $s = 4$ .

11. The dollar amount  $d$  that Megan earns varies directly with the number of hours  $h$  that she works. In her last paycheck working 18 hours, she earned \$90. If her next paycheck is \$90, how many hours did she work?

**Unit 8 Test Study Guide (Rational Functions)**

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Per: \_\_\_\_\_

**Topic 1: Simplifying Rational Expressions**

1.  $\frac{12x^2 - 30x}{20x^3 - 50x^2}$

2.  $\frac{4a^2 - 36}{24 - 8a}$

3.  $\frac{a^2 - 13a + 40}{3a^2 - 14a - 5}$

**Topic 2: Operations with Rational Expressions**

4.  $\frac{6p^2 - 13p + 5}{2p^2 + 17p - 9} \cdot \frac{p^2 + 16p + 63}{4p + 28}$

5.  $\frac{50 - 2w^2}{3w^2 + 9w - 30} \cdot \frac{w^2 + 5w - 14}{6w - 30}$

6.  $\frac{5y + 5}{2} \cdot \frac{25y - 20}{40y^2 - 32y}$

7.  $\frac{2c^2 + 4c - 6}{4c^2 - 7c + 3} \cdot \frac{16c^2 + 48c}{16c^2 - 9}$

8.  $\frac{6x}{x^2 - 16} \cdot \frac{x - 20}{x^2 - 16}$

9.  $\frac{16}{3} \cdot \frac{4k + 56}{3k + 15}$

Name: \_\_\_\_\_ Date: \_\_\_\_\_

Topic: \_\_\_\_\_ Class: \_\_\_\_\_

**GRAPHING Rational Functions**

**Use the key features**

- Write the function in FA
- Find the x-intercept(s)
- Find the vertical asymptote(s)
- Find the horizontal asymptote(s)

**CASE**

degree of  $p >$  degree

degree of  $p <$  degree

degree of  $p =$  degree

Identify any holes in the function.

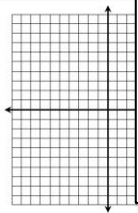
A hole is a point  $(x, y)$  at which there is a common factor in the numerator and denominator.

To find the x-coordinat \_\_\_\_\_

To find the y-coordinat \_\_\_\_\_

**Directions:** Graph each function and identify its key features.

1.  $f(x) = \frac{x - 4}{x + 4}$



Name: \_\_\_\_\_ Date: \_\_\_\_\_

Per: \_\_\_\_\_

**Unit 8 Test Rational Functions**

1. Simplify the expression below.

$\frac{2a^2 - 32a}{2a^3 + 16a + 32a}$

A.  $2(a - 4)$  C.  $\frac{(a - 4)}{(a + 4)}$

B.  $-2$  D.  $\frac{2(a - 4)}{(a + 4)}$

2. Simplify the expression below.

$\frac{4yz^2 - 15x^2y}{3xy^2z - 8x^3yz}$

A.  $\frac{5}{2x^2y^3}$  C.  $\frac{2x^2y^3}{5z}$

B.  $\frac{5z}{2x^2y^3}$  D.  $\frac{2x^2y^3}{5}$

3. Simplify the expression below.

$\frac{2 - 18w^2}{3w^2 - 8w - 3} \cdot \frac{3w^2 - 7w - 6}{12w^2 + 80w - 28}$

A.  $\frac{3w + 2}{2(w + 7)}$  C.  $\frac{-(3w + 2)}{2(w + 7)}$

B.  $\frac{3w + 2}{w + 7}$  D.  $\frac{-2(3w + 2)}{w + 7}$

4. Simplify the expression below.

$\frac{2c^2 - 5c - 3}{c^2 + 4c - 21} \cdot (2c + 1)$

A.  $\frac{2c + 1}{c + 7}$  C.  $\frac{(c + 3)}{(c + 7)(c - 3)}$

B.  $\frac{1}{c + 7}$  D.  $c + 7$

5. Simplify the expression below.

$\frac{p^2 - 14p + 40}{16p^2} \cdot \frac{3p^2 - 18p + 24}{2p^2 - 4p}$

A.  $\frac{24p^2}{p - 10}$  C.  $\frac{p - 10}{24p^2}$

B.  $\frac{24(p - 10)}{p^2}$  D.  $\frac{1}{24p^2(p - 10)}$

6. Simplify the expression below.

$\frac{17x - 9}{3x + 6} \cdot \frac{2x - 3}{3x + 6}$

A.  $-5x$  C.  $\frac{5x - 2}{x + 2}$

B.  $\frac{5x - 2}{3x}$  D.  $\frac{5x - 4}{x + 2}$